

# Model-Reference Adaptive Steering Control of a Farm Tractor with Varying Hitch Forces

J. Benton Derrick\* , David M. Bevly\* , Andrew K. Rekow†

\*Department of Mechanical Engineering  
Auburn University, AL 36849  
Email: {derrijb, dmbevly}@auburn.edu

†Deere & Company Tractor Works  
Waterloo, IA 50701  
Email: RekowAndrewK@johndeere.com

**Abstract**—This paper presents a model-reference adaptive controller (MRAC) for steering a farm tractor with varying hitch forces. Hitch forces play an important role in the yaw dynamics of the tractor, and these forces change with respect to different implements and soil conditions. It is desired that the tractor has the same dynamic response no matter what the hitch forces may be. A good dynamic yaw model has been developed for a tractor with hitch forces, and one parameter in this model changes with hitch loading. A MRAC algorithm is consequently proposed to directly adapt one controller parameter to changing hitch loading. A set of cascaded controllers will be used to regulate the tractor’s lateral position, although only the controller that directly regulates the yaw rate will be adapted. The adaptation algorithm is then augmented to account for inner-loop steering actuator dynamics and saturations. Simulated and experimental results are presented.

## I. INTRODUCTION

Research on control systems that regulate the position of ground vehicles has been done for the past couple of decades. Farm tractors are good candidates for being automatically steered since many maneuvers are continually repeated. Poor visibility, driver inexperience, driver fatigue, and overlap are all issues that can be addressed by automatically steered farm tractors. A John Deere 8420 production model with Starfire DGPS and AutoTrac technology is shown in Fig. 1. This tractor can track straight paths across fields with no driver input at the steering wheel.

Farm tractors can be outfitted with a myriad of different implements that encounter various soil conditions. These variations cause the tractor yaw dynamics to change over time. It is vital that the position controller keep the tractor on the desired path; crop destruction and/or uneven application of agricultural agents can result from a poorly designed position controller. The position controller on the tractor must be able to have a good dynamic response no matter what the configuration of the tractor may be. Therefore, an adaptive controller will be designed and implemented to adjust to varying dynamics.

Previously, there have been indirect self-tuning and estimation methods proposed to control tractors with parameter variations [1], [2]. These methods estimate yaw model



Fig. 1. John Deere 8420 Equipped with Starfire DGPS and AutoTrac Technology

parameters and adjust the controller parameters on-line to match a desired closed-loop system. An on-line method for directly adjusting the controller to changing hitch forces is desired and will be the focus of this paper. A good model of the yaw dynamics with hitch forces is known, and hitch forces account for the change in one model parameter [3]. This paper consequently utilizes a model-reference adaptive controller (MRAC) to directly update the controller parameters accounting for yaw plant parameter variations. A MRAC system was chosen so that the steering actuator saturations could be accounted for in the adaptation mechanism. In general, direct adaptation techniques adjust controller parameters so that the actual closed-loop performance converges to a desired closed-loop configuration [4]. Previous research has been done on MRAC of yaw dynamics dealing with active steering [5]. Also, vehicle guidance has been adapted using a form of MRAC [6]. A gain schedule approach to active steering of a ground vehicle has also been developed using a form of the bicycle model [7].

The tractor position controller is a set of cascaded controllers designed to regulate the steering angle, yaw rate, and lateral position. Since the yaw rate dynamics are a direct

function of hitch loading, the adaptation law will adjust the yaw rate controller to match the desired response. One inherit problem with this approach is the system's inner-loop steering actuator dynamics and non-linearities; they must be neglected or a higher-order adaptation algorithm must be used. A higher-order adaptation algorithm is undesirable since higher-order derivatives of the output are not measurable and differentiation tends to be noisy. If the actuator dynamics and non-linearities are neglected, this may result in the adaptation algorithm becoming unstable and/or giving poor performance. This paper will show that a lower-order adaptation law can be derived by augmenting the MRAC algorithm to account for these neglected properties.

## II. MODEL AND CONTROL STRUCTURE

### A. Tractor Dynamic Yaw Model

Previous research has been done on modeling the yaw dynamics of a tractor with hitch forces. O'Conner proposed a model where the hitch forces were modeled as an extra tire behind the rear axle [8]. Pearson studied the effects of hitch forces, and has shown the validity of O'Conner's model for implements attached to the tractor using the three-point hitch [3]. The yaw rate model of the tractor will provide some incite about how the yaw properties vary with hitch loading. This will then give a basis for why adaptive control is needed.

A bicycle model of a four-wheeled vehicle lumps the inner and outer tires together and neglects weight transfer. The bicycle model schematic with augmented hitch forces is shown in Fig. 2. One category of tire models assumes that the lateral forces at the tires ( $F_f$ ,  $F_r$ , and  $F_h$ ) are a function of the tire slip angles  $\alpha_f$ ,  $\alpha_r$ , and  $\alpha_h$  [9]. The linearized version of the bicycle model assumes that the lateral force at the tires are proportional to the slip angles by the coefficients  $C_{\alpha_f}$ ,  $C_{\alpha_r}$ , and  $C_{\alpha_h}$ . Lengths  $a$ ,  $b$ , and  $c$  are the distances from the front axle to the center of gravity, the rear axle to the center of gravity, and the rear axle to the center of force on the implement, respectively.  $V_y$  and  $V_x$  are the lateral and longitudinal velocities, and  $\delta$  is the steering angle of the front axle. By summing forces and moments and linearizing assuming small angles, the transfer function from steering angle to yaw rate ( $r$ ) of the bicycle model with lateral hitch forces is shown in (1).

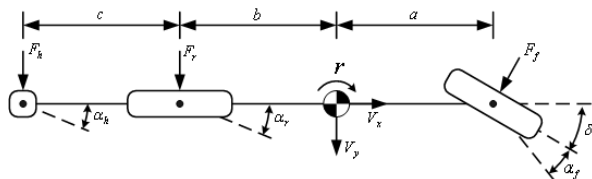


Fig. 2. Bicycle Model with Augmented Lateral Hitch Force

$$G_{Pr} = \frac{r(s)}{\delta(s)} = \frac{n_1 \cdot s + n_0}{d_2 \cdot s^2 + d_1 \cdot s + d_0}$$

$$\begin{aligned} n_0 &= \frac{C_{\alpha_f} C_1 + a C_{\alpha_f} C_2}{m V_x} \\ n_1 &= a C_{\alpha_f} \\ d_0 &= \frac{C_2 C_3 - C_1^2}{m V_x^2} + C_1 \\ d_1 &= \frac{C_2 I_{zz}}{m V_x} + \frac{C_3}{V_x} \\ d_2 &= I_{zz} \end{aligned} \quad (1)$$

$$\begin{aligned} C_1 &= ((b+c) \cdot C_{\alpha_h} + b \cdot C_{\alpha_r} - a \cdot C_{\alpha_f}) \\ C_2 &= (C_{\alpha_h} + C_{\alpha_r} + C_{\alpha_f}) \\ C_3 &= ((b+c)^2 C_{\alpha_h} + b^2 C_{\alpha_r} + a^2 C_{\alpha_f}) \end{aligned}$$

A kinematic relationship between yaw rate and lateral position is used. Lateral position ( $y$ ) can be described by (2) where  $\beta$  is the side slip angle at the center of gravity and  $\nu$  is the angle of the tractor's velocity vector with respect to the desired longitudinal direction.

$$\begin{aligned} \dot{y} &= V \sin(\nu) \\ \dot{\nu} &= r + \dot{\beta} \end{aligned} \quad (2)$$

The lateral position transfer function is given by (3).

$$G_{Py} = \frac{y(s)}{r(s)} = \frac{V}{s^2} \quad (3)$$

### B. Control Structure

A set of cascaded controllers is utilized for controlling the lateral position of the tractor. The cascaded control block diagram is shown in Fig. 3. Three feedback loops are implemented using the measurements from the steering angle sensor ( $\delta$ ), gyroscope ( $r$ ), and GPS receiver ( $y$ ). The lateral position, yaw rate, and steering angle controllers are noted by  $G_{Cy}$ ,  $G_{Cr}$ , and  $G_{C\delta}$ , respectively. The steering servo, yaw rate, and lateral position plants are noted by  $G_{P\delta}$ ,  $G_{Pr}$ , and  $G_{Py}$ , respectively.

A cascaded controller was chosen to make the controller design less complicated. The steering controller was chosen to have a proportional control law with the coefficient of  $k_{p\delta}$  shown in (4).

$$G_{C\delta} = \frac{\delta_{input}(s)}{\delta_{err}(s)} = k_{p\delta} \quad (4)$$

The yaw rate controller was also chosen to have a proportional control law with the coefficient of  $k_{pr}$  multiplied by the adaptation gain  $K$  shown in (5).

$$G_{Cr} = \frac{\delta_{des}(s)}{r_{err}(s)} = k_{pr} K \quad (5)$$

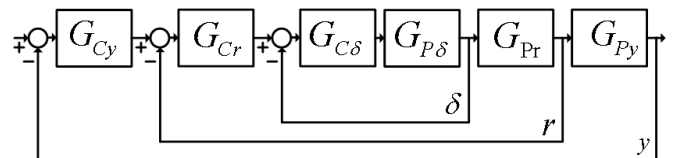


Fig. 3. Cascaded Control Block Diagram

The lateral position controller was chosen to have a proportional-integral-derivative (PID) control law shown in (6) with coefficients  $k_{py}$ ,  $k_{iy}$ , and  $k_{dy}$ .

$$G_{Cy} = \frac{r_{des}(s)}{y_{err}(s)} = k_{py} + \frac{k_{iy}}{s} + k_{dy} \cdot s \quad (6)$$

For the sake of this paper, velocity will be held constant. Therefore, the inner-loop steering controller and the outer-loop lateral position controller will be fixed-gain configurations. The yaw rate controller will be adapted to compensate for variations in hitch loading.

### III. MRAC ALGORITHM

It has been shown empirically by Gartley that the DC gain of the steering angle to yaw rate transfer function is the parameter that changes the most with hitch loading [2]. In (1), it can be seen that the varying parameter  $C_{\alpha h}$  changes more than just the DC gain; the poles and zero vary also. An assumption is now made that the parameter  $C_{\alpha h}$  affects the DC gain only and the effect on pole-zero location is negligible. Under these assumptions, it can be observed that scaling the loop gain of the yaw rate controller by  $K$  will provide an approximate model match to the bicycle model with a varying  $C_{\alpha h}$  parameter.

The MIT rule was used to derive a controller update law that will adapt the yaw rate control gain. The MIT rule was the original approach to MRAC [4]. The drawback of this method is that stability for a time varying system cannot be proven. Stability of linear time invariant systems using the MIT rule adaptation technique has been proven under certain circumstances [10], but the methods cannot be used since the tractor dynamics are time varying. A block diagram of the adaptive scheme is shown in Fig. 4.

The box labeled "Yaw Model" is the open-loop transfer function shown in (1) with a specified  $C_{\alpha h}$ . The value of  $C_{\alpha h}$  for the yaw model is chosen such that it is in the middle of the range that  $C_{\alpha h}$  can vary. This will be described in more detail in a later section. The box labeled "Yaw Plant" is the actual tractor plant. The section labeled "Yaw Rate Controller" is the controller that will be put in place of  $G_{Cr}$  in Fig. 3.

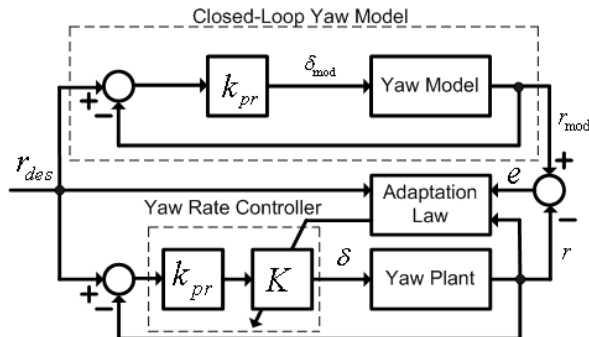


Fig. 4. MRAC for Yaw Rate Controller

By closing the loop with the proportional controller  $k_{pr}$  and adaptation gain  $K$ , the closed-loop yaw rate differential equation is shown to be (7).

$$r = \frac{1}{d_0 + k_{pr}K n_0} (k_{pr}K(n_1(\dot{r}_{des} - \dot{r}) + n_0 r_{des}) - d_1 \dot{r} - d_2 \ddot{r}) \quad (7)$$

The  $n_i$  and  $d_i$  coefficients are represented by the bicycle model coefficients in (1).

The adaptation algorithm is derived by minimizing the cost function of the error between the output of the model and the closed-loop system [4]. The cost function and error definition are shown in (8).

$$e = r_{mod} - r \quad (8)$$

$$J = \frac{1}{2} e^2$$

The MIT rule changes the controller parameter in the direction of the negative gradient of  $J$  with respect to  $K$ . The time rate of change of the adaptive controller parameter is represented by (9) that minimizes (8) with respect to the adaptation parameter  $K$ .

$$\frac{dK}{dt} = -\gamma \frac{\partial J}{\partial K} = -\gamma e \frac{\partial e}{\partial K} = \gamma e \frac{\partial r}{\partial K} \quad (9)$$

Applying the previous equations to (7), the adaptation algorithm of  $K$  is represented by (10).

$$\frac{dK}{dt} = \gamma \beta (n_1 d_0 (r_{des} - \dot{r}) + n_0 (d_0 r_{des} + d_1 \dot{r} + d_2 \ddot{r})) \cdot e \quad (10)$$

$$\beta = \frac{k_{pr}}{(d_0 + k_{pr}K n_0)^2}$$

Though some of the parameters in this equation are a function of the unknown parameter  $C_{\alpha h}$ , they can either be absorbed into the adaptation gain  $\gamma$  or approximated by using the same values as the reference yaw model. All that has to be known exactly is the sign of  $C_{\alpha h}$ .

### IV. MRAC MODIFICATIONS

#### A. Actuator Dynamics

Through simulation experiments, it was discovered that the adaptation will become unstable and/or perform poorly due to neglected steering actuator dynamics and non-linearities. The problem is solved in this case by including the actuator properties in the reference yaw model. The steering servo can be modeled as second order system with an integrator as seen in (11).

$$G_{P\delta} = \frac{\delta(s)}{\delta_{input}(s)} = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n \cdot s + \omega_n^2)} \quad (11)$$

Table I list the parameters identified using system identification experiments during previous research [2]. A block diagram of the new closed-loop reference model with steering actuator dynamics is shown in Fig. 5. This will replace the section in Fig. 4 that is labeled "Closed-Loop Yaw Model".

This modification is convenient in that the adaptive control update law does not change. This allows for a lower-order update law as compared with including the actuator dynamics

TABLE I  
STEERING ACTUATOR PARAMETERS

| Parameter  | Value       |
|------------|-------------|
| $\omega_n$ | 28.425rad/s |
| $\zeta$    | 0.633       |

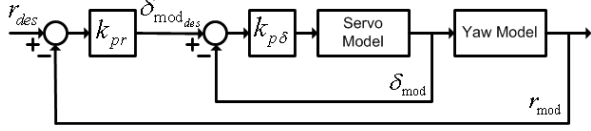


Fig. 5. MRAC Desired Closed-Loop Model with Steering Actuator Dynamics

into the adaptation algorithm. If the steering actuator dynamics are neglected, the model yaw rate will lead the tractor yaw rate and the two outputs will not converge to the same value.

### B. Actuator Saturations

Steering actuator saturations will cause the adaptation algorithm to yield poor performance and to exhibit instability. There are two different saturations that have to be accounted for:  $\delta_{max}$  and  $\dot{\delta}_{max}$  as seen in Table II. Both of these saturations will be accounted for by dictating the same restrictions on the model steering actuator. Therefore,  $\delta_{mod}$  will be held to (12).

$$\delta_{mod_{max}} = \delta_{max} \quad (12)$$

The hydraulic steering actuator can only turn the wheels at a maximum slew rate  $\dot{\delta}_{max}$ , and there after the actuator is saturated. If this saturation is ignored, the steering angle in the model goes to the desired set point more quickly than the steering angle of the tractor. This will destabilize the adaptation algorithm similarly to the wheel hitting the steering angle restraint. Therefore,  $\dot{\delta}_{mod}$  will be held to the same slew rate restrictions as  $\dot{\delta}$  as seen in (13).

$$\dot{\delta}_{mod_{max}} = \dot{\delta}_{max} \quad (13)$$

Through simulation experiments, it was found that holding the adaptation gain constant while the actuator is saturated will allow for a better gain response. Therefore, the adaptation gain update law will follow (14).

$$\frac{dK}{dt} = \begin{cases} \frac{dK}{dt} & \text{for } |\dot{\delta}_{meas}| < \dot{\delta}_{max} \quad |\delta_{meas}| < \delta_{max} \\ 0 & \text{for } |\dot{\delta}_{meas}| \geq \dot{\delta}_{max} \quad |\delta_{meas}| \geq \delta_{max} \end{cases} \quad (14)$$

TABLE II  
STEERING ACTUATOR SATURATION PARAMETERS

| Parameter            | Value     |
|----------------------|-----------|
| $\delta_{max}$       | 32deg     |
| $\dot{\delta}_{max}$ | 20.6deg/s |

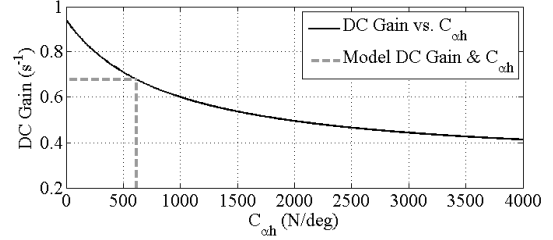


Fig. 6. DC Gain versus  $C_{ah}$  for the Bicycle Model with Augmented Hitch Force with  $V_x = 2\text{m/s}$

## V. SIMULATED AND EXPERIMENTAL RESULTS

### A. Simulated Results

It has been shown in previous research that  $C_{ah}$  can range between 0N/deg and 4000N/deg [3]. It is desired that the adaptation gain  $K$  have the value of 1 in the middle of the DC gain range; therefore,  $C_{ah}$  will be set to 600N/deg for the reference model as seen in Fig. 6.

A simulation assuming noise and bias free sensors was performed in MATLAB to prove that all of the assumptions and approximations mentioned in previous sections are valid. Table III lists the parameters used in the simulation. The tractor yaw rate plant was simulated with the same parameters as the reference model but with a different  $C_{ah}$  parameter. The simulation was conducted using a cosine function as the desired yaw rate. The cosine function was chosen so that the steering actuator will be initially saturated. Since one parameter is being adapted, one frequency of excitation will allow the controller parameter to reach its true value.

The adaptation gain  $K$  and yaw rate response is shown in Fig. 7. The desired gain is calculated by dividing the DC gain of the reference model by the DC gain of the simulated tractor. As can be seen, the adaptation gain  $K$  reaches the desired value, and the tractor and model yaw rates converge to the same value. This shows that just adapting the proportional control gain of the yaw rate controller will provide a good match to a model with a different  $C_{ah}$  parameter.

The steering actuator response is shown in Fig. 8. By observing the slew rate response, it can be seen that the

TABLE III  
TRACTOR MODEL PARAMETERS

| Parameter            | Value                    |
|----------------------|--------------------------|
| $a$                  | 1.00m                    |
| $b$                  | 2.00m                    |
| $c$                  | 2.19m                    |
| $I_{zz}$             | 18500kg · m <sup>2</sup> |
| $m$                  | 11340kg                  |
| $C_{\alpha f}$       | 2400N/deg                |
| $C_{\alpha r}$       | 5000N/deg                |
| $C_{\alpha h, mod}$  | 600N/deg                 |
| $C_{\alpha h, trac}$ | 4000N/deg                |
| $V_x$                | 2m/s                     |
| $k_{pr}$             | 0.40                     |
| $k_{p\delta}$        | 3.84                     |

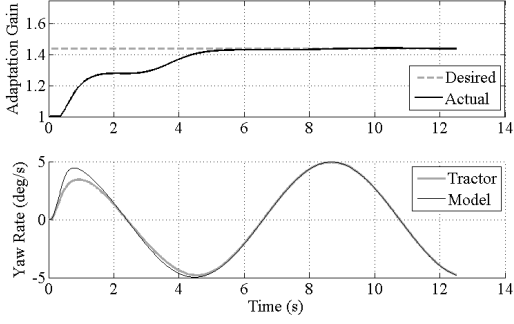


Fig. 7. Simulated Adaptation Gain & Yaw Rate Response

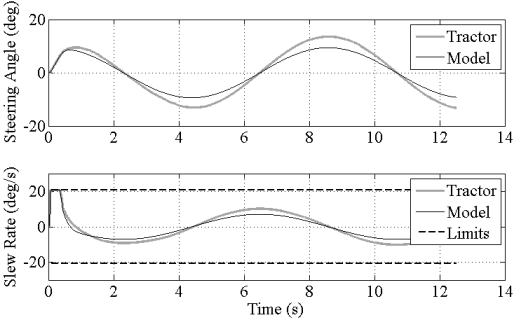


Fig. 8. Simulated Steering Actuator Response

slew rate was initially saturated. By looking at Fig. 7, it can be seen that the adaptation gain was held constant at  $K = 1$  for the duration of the saturated period. This proves that the adaptation law works when the steering actuator is allowed to saturate. Since the tractor in the simulation has a greater  $C_{\alpha h}$  parameter, more steering angle is required to achieve the same yaw rate as the model. This can be seen by looking at the steering angle plot in Fig. 8. The tractor and the model both have the same steering angle initially, but the tractor has more steering angle when the adaptation gain increases.

### B. Experimental Results

The algorithm was next implemented on the John Deere 8420. For this experiment, the tractor will track a straight line. The tractor is guided by commanding a flow rate to the hydraulic steering valve at 50 Hz. The GPS receiver and lateral position loop was executed at 5 Hz, and the other measurement and control loops were executed at 50 Hz. To combat gyroscopic sensor noise, a second-order Butterworth filter at 5 Hz was implemented on the measured yaw rate.

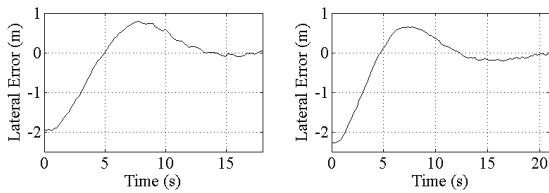


Fig. 9. Experimental Lateral Error Response (Left Column: 4 Shank Ripper; Right Column: No Implement)

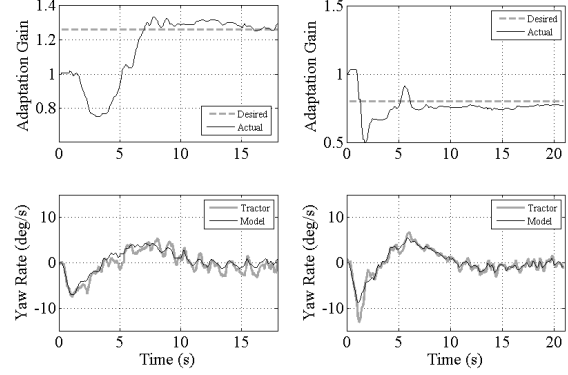


Fig. 10. Experimental Adaptation Gain & Yaw Rate Response (Left Column: 4 Shank Ripper; Right Column: No Implement)

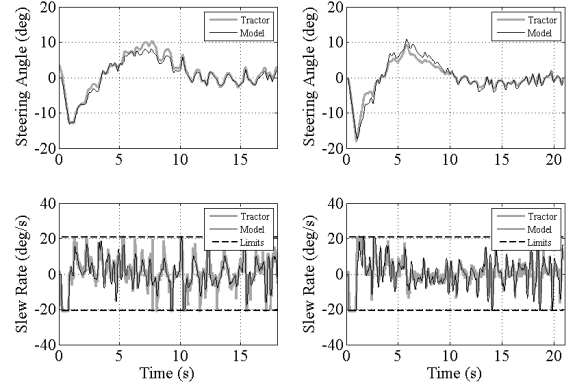


Fig. 11. Experimental Steering Actuator Response (Left Column: 4 Shank Ripper; Right Column: No Implement)

Two different experiments are presented. The first experiment was performed with a four-shank ripper attached to the three-point hitch, and the second was performed with no implement. In both experiments, the tractor was initially driving at a constant  $V_x \approx 2.0\text{m/s}$  with an initial value of  $y_{err} \approx 2.0\text{m}$ . Fig. 9 shows the lateral error response of the tractor with (Left Column) and without (Right Column) an implement.

The adaptation gain  $K$  and yaw rate response is shown in Fig. 10. It can be seen in the adaptation gain plot that the actual gain reaches close to the desired value. The desired value for the adaptation was determined in previous experiments. The actual gain doesn't reach the true value because the yaw rate was not persistently excited for a long enough period of time. The yaw rate is noisy due to the rough farm land that the experiment was performed on. A higher loop-gain of the yaw rate controller makes the system more sensitive to sensor noise and ground disturbances. This increased sensitivity can be seen in the steering angle response in Fig. 11. The higher adaptation gain increases the loop-gain of the yaw rate controller and makes the steering angle oscillate slightly. Because the DC gain of the tractor decreases with increasing  $C_{\alpha h}$ , more steering angle is required to reject disturbances.

The steering actuator response is shown in Fig. 11. As in the simulation, the slew rate of the steering angle becomes initially saturated during the maneuver. The adaptation gain  $K$  was therefore held constant during this period. It can be seen that the gain is allowed to adapt once the slew rate becomes unsaturated. Therefore, the algorithm is shown to work with steering actuator saturations.

## VI. CONCLUSIONS AND FUTURE WORKS

### A. Conclusions

A model-reference adaptive steering controller has been presented that adjusts to changing hitch loading on a farm tractor. The cascaded control structure was presented that regulates the lateral position of the farm tractor. A parametric model was shown that predicts how hitch loading effects yaw dynamics, and this model is used as the reference-model in the adaptation scheme. The MIT rule gradient approach was used to derive an update law for the adaptation parameter, and the reference model was augmented with steering actuator dynamics and saturations. It has been shown through simulation and experimentation that the MRAC algorithm is sufficient in adjusting the yaw rate controller to compensate for changing hitch loading conditions.

### B. Future Works

This paper has demonstrated the preliminary results of this research. Noise and biases degrade the performance of the algorithm; therefore, future work should include a method of reducing the sensor imperfections. A Kalman filter should be developed to reduce the noise and estimate the yaw rate gyroscope bias using the course measurement from the GPS receiver. A method should be developed to decrease the steering oscillation due to the increasing loop gain required to control the tractor with large implements. It would also be desirable to have a MRAC that will adapt while the steering actuator is saturated instead of holding the adaptation gain constant.

## VII. ACKNOWLEDGMENTS

The authors greatly appreciate Deere & Company's support of this research including the use of a John Deere 8420 tractor and a Starfire DGPS receiver. Special thanks to Greg Pate of the E.V. Smith Research Center for accommodating the experimental testing of this research. Also, thanks goes to Dr. Randy Raper of the USDA National Soil Dynamics Laboratory for the use of an implement.

## REFERENCES

- [1] A. Rekow, "System identification, adaptive control and formation driving of farm tractors," Ph.D. dissertation, Department of Aeronautics and Astronautics, Stanford University, March 2001.
- [2] E. Gartley and D. Bevy, "On-line adaptive control of a farm tractor by compensations of parameter variations," in *Proceedings of the IMECE Conference, Orlando, FL, 2005*.
- [3] P. Pearson and D. Bevy, "Comparison of analytical and empirical models to capture variations in off road vehicle dynamics," in *Proceedings of the IMECE Conference, Orlando, FL, 2005*.
- [4] K. Astrom and B. Wittenmark, *Adaptive Control*, 2nd ed. Addison Wesley, Reading, MA, 1995.

- [5] T. Fukao, "Active steering systems based on model reference adaptive nonlinear control," in *Proceedings of the IEEE Intelligent Transportation Systems Conference, Oakland, CA, August 2001*.
- [6] T. Hessburg, "Model reference adaptive fuzzy logic control for vehicle guidance," in *Proceedings of the American Control Conference, Seattle, WA, June 1995*.
- [7] S. Baslamish, "Gain scheduled active steering control based on a parametric bicycle model," in *Proceedings of the IEEE Intelligent Vehicles Symposium, June 2007*.
- [8] M. O'Conner, "Carrier-phase differential gps for automatic control of land vehicles," Ph.D. dissertation, Department of Aeronautics and Astronautics, Stanford University, December 1997.
- [9] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers, Warrendale, PA, 1992.
- [10] I. Mareels, "Global stability for an mit rule adaptive control algorithm," in *Proceedings of the 28th Conference on Decision and Control, December 1989*.

TABLE IV  
NOMENCLATURE TABLE OF VARIABLES USED IN PAPER

| Variable                 | Definition                               |
|--------------------------|--|
| $a$                      | Distance From CG to Front Axle           |
| $b$                      | Distance From CG to Rear Axle            |
| $c$                      | Distance From Rear Axle to Hitch         |
| $I_{zz}$                 | Mass Moment of Inertia About the CG      |
| $m$                      | Mass                                     |
| $C_{\alpha f}$           | Cornering Stiffness of Front Axle        |
| $C_{\alpha r}$           | Cornering Stiffness of Rear Axle         |
| $C_{\alpha h}$           | Cornering Stiffness of the Hitch         |
| $\alpha f$               | Slip Angle at Front Axle                 |
| $\alpha r$               | Slip Angle at Rear Axle                  |
| $\alpha h$               | Slip Angle at Hitch                      |
| $F_f$                    | Lateral Force at Front Axle              |
| $F_r$                    | Lateral Force at Rear Axle               |
| $F_h$                    | Lateral Force at Hitch                   |
| $V$                      | Total Velocity                           |
| $V_x$                    | Longitudinal Velocity                    |
| $V_y$                    | Lateral Velocity                         |
| $\delta$                 | Tractor Steering Angle                   |
| $\delta_{err}$           | Tractor Steering Angle Error             |
| $\delta_{mod}$           | Model Steering Angle                     |
| $\dot{\delta}$           | Tractor Steering Angle Rate              |
| $\delta_{input}$         | Steering Input Command                   |
| $\delta_{des}$           | Desired Steering Angle                   |
| $\delta_{max}$           | Maximum Steering Angle                   |
| $\dot{\delta}_{max}$     | Maximum Steering Angle Rate              |
| $r$                      | Tractor Yaw Rate                         |
| $r_{err}$                | Tractor Yaw Rate Error                   |
| $r_{mod}$                | Model Yaw Rate                           |
| $r_{des}$                | Desired Yaw Rate                         |
| $y$                      | Lateral Position                         |
| $y_{err}$                | Lateral Position Error                   |
| $G_{P\delta}$            | Steering Actuator Plant                  |
| $G_{Pr}$                 | Yaw Rate Plant                           |
| $G_{Py}$                 | Lateral Position Plant                   |
| $G_{C\delta}$            | Steering Actuator Controller             |
| $G_{Cr}$                 | Yaw Rate Controller                      |
| $G_{Cy}$                 | Lateral Position Controller              |
| $n_i, d_i, C_i$          | Yaw Rate Model Coefficients              |
| $k_{py}, k_{dy}, k_{iy}$ | Lateral Position Controller Coefficients |
| $k_{pr}$                 | Yaw Rate Controller Coefficient          |
| $k_{p\delta}$            | Steering Actuator Controller Coefficient |
| $K$                      | Adaptation Gain                          |
| $e$                      | Adaptation Error                         |
| $J$                      | Adaptation Cost Function                 |
| $\gamma$                 | Adaptation Algorithm Gain                |
| $\omega_n$               | Steering Actuator Natural Frequency      |
| $\zeta$                  | Steering Actuator Damping Ratio          |
| $\beta$                  | Slip Angle at Center of Gravity          |
| $\nu$                    | Angle Between Velocity and Desired Path  |