Split-Gate Tracking Accuracy For Phase Coded CW Radar

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Abstract—In this paper a phase coded continuous wave (CW) radar system is analyzed. The motivation for the research was to accelerate tracking filter convergence. The main goal of the paper is to express the variance of the range errors as a function of signal to noise ratio and position in the split-gate region. Furthermore, the effect of different signal processing algorithms on the variance of the range errors is investigated. The effects of channel mismatch in the receiver and range gate spacing on the variance of the range errors are also included. Analytical equations that relate the variance of the range errors to signal to noise ratio (SNR) and position within the split gate region are derived using a Taylor series expansion. The analytical equations are consistent with a statistical analysis of a simulation of the radar system in MATLAB. The results of the work are the analytical equations for the variance of the range error as a function of SNR, range gate spacing, and channel mismatch for a split-gate tracker.

I. INTRODUCTION

The radar system analyzed in this paper was developed at Phase IV Systems in Huntsville, AL. The radar system was designed to be mounted on a ground vehicle and track targets approaching the vehicle. The system measures the distance from the radar face to the target by using the autocorrelation properties of maximal length codes. These measurements are input to a Kalman filter which produces refined range estimates. The purpose of this research is to develop a range variance expression in the form of equation (1) for split gate tracking.

$$\sigma_{target} = k(\Delta m, \Delta x) \frac{\delta R}{\sqrt{SNR}} \tag{1}$$

In equation (1), δR is the spacing of the range gates in a split-gate track, SNR is the Signal to Noise Ratio, and $k(\Delta m, \Delta x)$ is a scaling factor which is a function of position, Δx , and channel mismatch, Δm , in the receiver. The paper examines the noise induced range errors inherent in a splitgate range track. One goal of the research is to determine whether the variance of the range measurement errors is a function of position within the split-gate region. A second goal of the paper is to develop a relationship between the variance of the range measurement errors and the signal to noise ratio of the returned signal and channel mismatch. The third goal of the report is to compare the performance of three different discriminator functions. Knowledge of how these factors corrupt the range measurements leads to better modeling of the range errors. More accurate modeling of the errors improves the performance of the tracking filters [1]. The variance of the range errors as a function of SNR, position between the range gates, and channel mismatch is derived mathematically for two different discriminator functions. A simulation of a target passing through the range gates is used to validate the empirical formulas and examine a third discriminator function.

Split-gate tracking is well understood and prior work has investigated the effects of noise on range accuracy. However, the standard deviation of the range errors in previous work is assumed to be unrelated to the target's position in the split-gate region. For Pulse-Doppler systems, several radar textbooks provide equations for calculating the RMS value of the range errors, [2], [3], [4]. None of these sources consider the target's position to be an influence on the range accuracy of the radar. Most radar textbooks do not analyze CW split-gate trackers in sufficient detail to provide equations for their accuracy. CW split-gate tracking is analyzed in great detail in the study of GPS (Global Positioning System). The discriminator functions analyzed in this paper fall into a category of discriminators referred to in GPS literature as the normalized early-minus-late power discriminators. Equations have been published which describe the range variance of this type of code discriminator in relation to SNR, [5]. The equations do not consider the offset between the actual and predicted phase of the received code. This offset in phase between the received and locally generated GPS codes is equivalent to a target moving through the split-gate region.

II. SYSTEM DESCRIPTION

The signal emitted by the radar is modeled as (2). In (2), A_e is the amplitude of the signal and ω_c is its radian frequency. G(t) is a maximal length sequence consisting of $\pm 1's$.

$$S_e(t) = A_e G(t) \cos(\omega_c t) \tag{2}$$

The signal is broadcast and then reflected back by the incoming target. Figure 1 shows a block diagram of the radar. The signal is passed through the receiver front-end and is modeled as (3). In (3), A_r is the amplitude of the returned signal after passing through the front-end, ω_d is the Doppler

shift in frequency caused by the target's velocity, and τ is the time delay between when the signal is emitted and when it is received.

$$S_r(t) = A_r G(t-\tau) \cos((\omega_c + \omega_d)(t-\tau))$$
(3)



Fig. 1. Block Diagram of Radar System

After passing through the receiver, the signal S_r is split three ways and mixed with three different LO signals. Each LO signal is a delayed version of the transmitted signal. Equations (4) through (6) represent the three delayed versions of the broadcast signal. In equations (4) through (6), τ_a , τ_b , and τ_c are fixed time delays. The term ω_{IF} represents the Intermediate Frequency (IF) of the radar system.

$$S_a(t) = G(t - \tau_a)\cos((\omega_c - \omega_{IF})t)$$
(4)

$$S_b(t) = G(t - \tau_b) \cos((\omega_c - \omega_{IF})t)$$
(5)

$$S_c(t) = G(t - \tau_c)\cos((\omega_c - \omega_{IF})t)$$
(6)

In the ideal case, the signal produced by mixing signals (4) and (3) is modeled as (7). Trigonometric identities are used to rearrange (7) into (8), [6].

$$M_{a}(t) = A_{r}G(t-\tau)G(t-\tau_{a})$$

$$\cos((\omega_{c}-\omega_{IF})t)$$

$$\cos((\omega_{c}+\omega_{d})(t-\tau))$$
(7)

$$M_{a}(t) = A_{r}G(t-\tau)G(t-\tau_{a})$$

$$[\cos((\omega_{IF}+\omega_{d})t) \qquad (8)$$

$$+\cos((2\omega_{c}-\omega_{IF}+\omega_{d})t)$$

$$-(\omega_{c}+\omega_{d})(\tau))]$$

The signal $M_a(t)$ is then bandpass filtered about ω_{IF} . This filtering removes the high frequency bracketed term in (8). The filter bandwidth is designed to pass the expected range of Doppler frequencies. The signals $G(t - \tau)$ and $G(t - \tau_a)$ both have a power spectral density consisting of discrete line components, [7]. The signal produced by multiplying them consequently has a discrete spectrum. The bandpass filter removes all the spectral lines except the centerline. The amplitude of the centerline is proportional to the time average of the product of the two maximal length sequences. Equation (9) expresses the output of the bandpass filter in the frequency domain. Here, T is the period of the code.

$$V_a(\omega) = A_r \frac{1}{T} \left[\int_0^T G(t-\tau) G(t-\tau_a) \, dt \right] \delta(\omega - \omega_{IF} - \omega_d)$$
(9)

The integral in (9) is the definition of autocorrelation. Equation (10) expresses the output of the bandpass filter in the frequency domain as the autocorrelation of the maximal length code offset from the IF frequency by the Doppler shift ω_d . The outputs of the other two mixers are similarly filtered and can be expressed as equations (11) and (12).

$$V_a(\omega) = A_r R(\tau_a - \tau) \delta(\omega - \omega_{IF} - \omega_d)$$
(10)

$$V_b(\omega) = A_r R(\tau_b - \tau) \delta(\omega - \omega_{IF} - \omega_d)$$
(11)

$$V_c(\omega) = A_r R(\tau_c - \tau) \delta(\omega - \omega_{IF} - \omega_d)$$
(12)

Following the bandpass filter, the signal is sampled and a Discrete Fourier Transform (DFT) is performed on the sampled signal. In the DFT, the centerline will appear offset from the IF frequency by an amount equal to the Doppler shift on the received signal. The amplitude of the centerline is proportional to the correlation between the received and locally generated signals. The amplitudes of the centerlines from each of the three channels are referred to as V_a , V_b , and V_c . The three amplitudes are processed to determine the targets's distance.



Fig. 2. Autocorrelation Function of Maximal Length Code

Figure 2 shows the autocorrelation function of a maximal length code. The autocorrelation function has a peak at $\tau = 0$ and decreases linearly to $-\frac{1}{L}$ at $\tau = \pm 1$, where L is the length of the code. Another attractive attribute of the maximal length code is that the range side lobes remain constant for all shifts, except when the codes align. This is significant because codes that do not have constant range side lobes may cause false alarms in the detection process. The delay τ_a is chosen such that at distance R_{rga} the autocorrelation function is at a maximum. As the target approaches this

distance from the radar, the amplitude of V_a in (10) increases linearly with distance. The increase in amplitude is referred to as correlating up. After passing R_{rga} , V_a decreases linearly with the approaching distance of the target. This decrease in amplitude is referred to as correlating down. Similarly, two distances, R_{rgb} and R_{rgc} , exist where V_b and V_c are at a maximum, respectively. The regions where V_a , V_b , and V_c overlap are referred to as the split-gate regions.



Fig. 3. Overlapping Range Gates

Figure 3 shows the overlapping range gates. R_{rga} is the shortest distance from the radar face and R_{rgc} is the furthest. When the target is between R_{rga} and R_{rgc} , the radar is configured for two different split-gate tracking regions. The two highest amplitudes are used to determine the target's distance.

When the target is between R_{rgc} and R_{rgb} , equation (13) is used to compute the target's distance. In equation (13), δR is the distance between the peaks of the different autocorrelation functions.

$$R_{target} = \frac{L+1}{L-1} \frac{V_c - V_b}{V_c + V_b} \frac{\delta R}{2} + R_{rgb} + \frac{\delta R}{2}$$
(13)

If the target is between R_{rga} and R_{rgb} , V_c and V_b are replaced with V_b and V_a , respectively. For the rest of this paper only the scenario where the target is between R_{rgc} and R_{rgb} will be considered, since the results would be identical for a target between R_{rgb} and R_{rga} .

III. ERROR MODELING

Two sources of error are considered to corrupt the range measurement R_{target} . The first is narrow band noise that is added by the receiver front-end. The second source is channel mismatch in the different receiver correlator outputs. The signal leaving the receiver is now modeled as equation (14). In (14), x(t) and y(t) are the in-phase and quadrature components of the noise added by the front-end [8], respectively. The noise components x(t) and y(t) are both bandlimited Gaussian processes.

$$S_r(t) = A_r G(t-\tau) \cos((\omega_c + \omega_d)(t-\tau)) + x(t) \cos(\omega_c t) + y(t) \cos(\omega_c t)$$
(14)

As in equation (7), the signal is mixed with three delayed versions of the emitted signal in equations (15) through (17).

Each delayed signal now has a gain associated with it to represent the channel mismatch.

$$S_a(t) = A_a G(t - \tau_a) \cos((\omega_c - \omega_{IF})t)$$
(15)

$$S_b(t) = A_b G(t - \tau_b) \cos((\omega_c - \omega_{IF})t)$$
(16)

$$S_c(t) = A_c G(t - \tau_c) \cos((\omega_c - \omega_{IF})t)$$
(17)

The signal leaving the *a* channel mixer is modeled as (18). The signal leaving each mixer is then bandpass filtered about ω_{IF} and the high frequency terms are eliminated.

$$M_{a}(t) = A_{r}A_{a}G(t-\tau)G(t-\tau_{a})\cos((\omega_{IF}+\omega_{d})t) +A_{a}x(t)G(t-\tau_{a})\cos(\omega_{IF}t)$$
(18)
+A_{a}y(t)G(t-\tau_{a})\sin(\omega_{IF}t) +
higher frequency terms

The signal leaving the bandpass filter is now modeled in the frequency domain as (19). The asterisk has been added to V_a^* to denote that the measurement has been corrupted by noise, not to signify conjugation. The Doppler frequency is assumed to be zero in (19). For the rest of the paper, the Doppler shift will be assumed to be zero. This assumption does not effect the validity our results.

$$V_a^*(\omega) = A_a[A_r R(\tau_a - \tau) + E[x(t)G(t - \tau_a)] + jE[y(t)G(t - \tau_a)]]\delta(\omega - \omega_{IF})$$
(19)

The signal V_a^* can be rewritten as the true amplitude V_a plus the contribution of the error terms $\delta_{x,a}$ and $\delta_{y,a}$ caused by the correlation of the noise and the delayed code, (20). The voltages produced by the *b* and *c* channel correlators can be similarly written, (21), (22).

$$V_a^*(\omega) = A_a[V_a + \delta_{x,a} + j\delta_{y,a}]$$
⁽²⁰⁾

$$V_b^*(\omega) = A_b[V_b + \delta_{x,b} + j\delta_{y,b}]$$
(21)

$$V_c^*(\omega) = A_c[V_c + \delta_{x,c} + j\delta_{y,c}]$$
(22)

IV. NOISE EFFECTS ON RANGING PRECISION

The values of V_a^* , V_b^* , and V_c^* are provided by a DFT operation on sampled data from each of the correlator channels. Equation (23) is the general method used to transform the values V_a^* , V_b^* , and V_c^* into range measurements. The values provided by the DFT are complex, but the target's range must be a real number. Three discriminators are considered, each transforms the complex data from the DFT into a real range measurement differently.

$$R_{target} = \frac{L+1}{L-1} \frac{V_c^* - V_b^*}{V_c^* + V_b^*} \frac{\delta R}{2} + R_{rgb} + \frac{\delta R}{2}$$
(23)

A. First Discriminator

The first method uses only the real component's of V_b^* and V_c^* in equation (23). The result is equation (24).

$$R_{target} = \frac{L+1}{L-1} \frac{A_c[V_c + \delta_{x,c}] - A_b[V_b + \delta_{x,b}]}{A_c[V_c + \delta_{x,c}] + A_b[V_b + \delta_{x,b}]} \frac{\delta R}{2} + R_{rgb} + \frac{\delta R}{2}$$
(24)

To derive the variance of R_{target} in (24), the division in (24) is approximated with a first order Taylor Series expanded around the error terms, (25).

$$\frac{A_c[V_c + \delta_{x,c}] - A_b[V_b + \delta_{x,b}]}{A_c[V_c + \delta_{x,c}] + A_b[V_b + \delta_{x,b}]} = f(\delta_{x,c}, \delta_{x,b}) \quad (25)$$

where

$$f(\delta_{x,c}, \delta_{x,b}) \approx f(0,0) + \delta_{x,c} \frac{\partial f(\delta_{x,c}, \delta_{x,b})}{\delta_{x,c}} \bigg|_{\delta_{x,b}=0,\delta_{x,c}=0} + \delta_{x,b} \frac{\partial f(\delta_{x,c}, \delta_{x,b})}{\delta_{x,b}} \bigg|_{\delta_{x,b}=0,\delta_{x,c}=0}$$

Carrying out the differentiation in (25) results in (26).

$$f(\delta_{x,c}, \delta_{x,b}) \approx \frac{A_c V_c - A_b V_b}{A_c V_c + A_b V_b}$$
(26)
+ $\delta_{x,c} \frac{2A_c A_b V_b}{(A_c V_c + A_b V_b)^2} - \delta_{x,b} \frac{2A_c A_b V_c}{(A_c V_c + A_b V_b)^2}$

Equation (27) shows the division in (24) replaced with the first order Taylor series.

$$R_{target} = \frac{L+1}{L-1} \left\{ \frac{A_c V_c - A_b V_b}{A_c V_c + A_b V_b} + \delta_{x,c} \frac{2A_c A_b V_b}{(A_c V_c + A_b V_b)^2} - \delta_{x,b} \frac{2A_c A_b V_c}{(A_c V_c + A_b V_b)^2} \right\} \frac{\delta R}{2} + R_{rgb} + \frac{\delta R}{2} \quad (27)$$

The variance of the error in the range measurements is defined in (28). In (28), $\overline{R_{target}}$ is the mean value of equation (27). Substituting equation (27) into (28) produces (29). The expected values of $\delta_{x,c}^2$ and $\delta_{x,b}^2$ and the expected value of $\delta_{x,c} \delta_{x,b}$ must be computed to use equation (29).

$$\sigma_{target}^2 = E[(R_{target} - \overline{R_{target}})^2]$$
(28)

$$\sigma_{target}^{2} = \left(\frac{L+1}{L-1}\right)^{2} \left(\frac{\delta R}{2}\right)^{2} E \left[\delta_{x,c}^{2} \frac{4A_{c}^{2}A_{b}^{2}V_{b}^{2}}{(A_{c}V_{c}+A_{b}V_{b})^{4}} + \delta_{x,b}^{2} \frac{4A_{c}^{2}A_{b}^{2}V_{c}^{2}}{(A_{c}V_{c}+A_{b}V_{b})^{4}} - \delta_{x,c}\delta_{x,b} \frac{8A_{c}^{2}A_{b}^{2}V_{c}V_{b}}{(A_{c}V_{c}+A_{b}V_{b})^{4}}\right]$$
(29)

The definition $\delta_{x,c}$ is used to derive its expected value squared, (30). The product of the two integrals in (30) can be rearranged into a double integral, (31). The integrals in (31) are carried out with respect to t_1 and t_2 .

$$E[\delta_{x,c}^2] = E\left[\left(\frac{1}{T}\int_0^T x(t)G(t-\tau_c)dt\right) \\ \left(\frac{1}{T}\int_0^T x(t)G(t-\tau_c)dt\right)\right]$$
(30)

$$\int_{0}^{T} x(t)G(t-\tau_{c})dt \int_{0}^{T} x(t)G(t-\tau_{c})dt = \int_{0}^{T} \int_{0}^{T} x(t_{1})x(t_{2})G(t_{1}-\tau_{c})G(t_{2}-\tau_{c})dt_{1}dt_{2}$$
(31)

The expectation operator and integration are linear functions. The expectation operator can therefore be moved inside the double integral. The noise x(t) and maximal length code are uncorrelated so their expected values can be computed separately, (32). The expected value of the product of a sequence and a copy of itself at a different time is the autocorrelation of the sequence. Equation (33) shows the expectations in (32) replaced with the noise and code autocorrelation functions. The noise x(t) is assumed to sufficiently high in bandwidth that its autocorrelation function can be modeled as a delta function.

$$E[\delta_{x,c}^{2}] = \frac{1}{T^{2}} \left[\int_{t_{1}=0}^{T} \int_{t_{2}=0}^{T} E[x(t_{1})x(t_{2})] \\ E[G(t_{1}-\tau_{c})G(t_{2}-\tau_{c})]dt_{1}dt_{2} \right]$$
(32)

$$E[\delta_{x,c}^2] = \frac{1}{T^2} \int_{t_1=0}^T \int_{t_2=0}^T \sigma_x^2 \delta(t_1 - t_2) R(t_1 - t_2) dt_1 dt_2$$
(33)

The delta function inside (33) is zero whenever t_1 and t_2 are not equal. When t_1 and t_2 are equal R(0) is one and the inner integral is equal to σ_x^2 , (34). The expected value of $\delta_{x,c}$ is therefore displayed in (35). The expected value of $\delta_{x,b}$ is similarly derived and shown in (36).

$$E[\delta_{x,c}^2] = \frac{1}{T^2} \int_{t_1=0}^T \sigma_x^2 dt_1$$
(34)

$$E[\delta_{x,c}^2] = \frac{\sigma_x^2}{T} \tag{35}$$

$$E[\delta_{x,b}^2] = \frac{\sigma_x^2}{T} \tag{36}$$

The expected value of the quantity $\delta_{x,c}\delta_{x,b}$ can be derived in a manner similar to that shown for $E[\delta_{x,c}^2]$, (37). In Equation (37), the inside integral evaluates to $R(\tau_c - \tau_b)$ when t_1 and t_2 are equal and zero otherwise. The expected value of $\delta_{x,c}\delta_{x,b}$ is therefore shown in (38).

$$E[\delta_{x,c}\delta_{x,b}] = \frac{1}{T^2} \int_{t_1=0}^T \int_{t_2=0}^T \sigma_x^2 \delta(t_1 - t_2) \cdot R(t_1 - t_2 + \tau_c - \tau_b) dt_1 dt_2$$
(37)

$$E[\delta_{x,c}\delta_{x,b}] = \frac{\sigma_x^2 R(\tau_c - \tau_b)}{T}$$
(38)

Equation (39) shows the expectations in equation (29) replaced with their actual values.

$$\sigma_{target}^{2} = \left(\frac{L+1}{L-1}\right)^{2} \left(\frac{\delta R}{2}\right)^{2} \left[\frac{4A_{c}^{2}A_{b}^{2}V_{b}^{2}}{(A_{c}V_{c}+A_{b}V_{b})^{4}} + \frac{4A_{c}^{2}A_{b}^{2}V_{c}^{2}}{(A_{c}V_{c}+A_{b}V_{b})^{4}} - \frac{8R(\tau_{c}-\tau_{b})A_{c}^{2}A_{b}^{2}V_{c}V_{b}}{(A_{c}V_{c}+A_{b}V_{b})^{4}}\right] \left(\frac{\sigma_{x}^{2}}{T}\right)$$
(39)

The variance of the range errors can be related to signal to noise ratio by remembering the definition of the voltages V_a , V_b , and V_c ,(40). Each term contains the amplitude of the received signal after passing through the receiver's front-end. SNR is defined in equation (41) for a sinusoid as the ratio of the signal power to the noise power, P_n . For narrowband noise, the noise power is equal to the variance of x(t) and to the variance of y(t), [8]. The SNR level at the input to the mixer can therefore be expressed as the ratio of the signal power, $\frac{1}{2}A^2$, to the noise power, σ_x^2 or σ_y^2 .

$$V_a = A_r R(\tau_a - \tau) = A_r R_a$$

$$V_b = A_r R(\tau_b - \tau) = A_r R_b$$

$$V_c = A_r R(\tau_c - \tau) = A_r R_c$$
(40)

$$SNR = \frac{\frac{1}{2}A^2}{P_n}$$

$$P_n = \sigma_x^2 = \sigma_y^2$$

$$SNR = \frac{\frac{1}{2}A^2}{\sigma_x^2} = \frac{\frac{1}{2}A^2}{\sigma_y^2}$$
(41)

Replacing the terms V_b and V_c in (39) with their definitions in (40) and rearranging yields equation (42). The definition of SNR can now be used to express the variance of the range errors as a function of SNR, channel mismatch, and position within the split-gate region, (43). Equation (43) can be arranged into the form of equation (44).

$$\sigma_{target}^{2} = \left(\frac{L+1}{L-1}\right)^{2} \left(\frac{\delta R}{2}\right)^{2} \left[\frac{4A_{c}^{2}A_{b}^{2}R_{b}^{2}}{(A_{c}R_{c}+A_{b}R_{b})^{4}} + \frac{4A_{c}^{2}A_{b}^{2}R_{c}^{2}}{(A_{c}R_{c}+R_{b}R_{b})^{4}} - \frac{8R(\tau_{c}-\tau_{b})A_{c}^{2}A_{b}^{2}R_{c}R_{b}}{(A_{c}R_{c}+A_{b}R_{b})^{4}}\right] \frac{\sigma_{x}^{2}}{A_{r}^{2}T}$$

B. Second Discriminator

The second discriminator function is shown in (45). Here, all the operations inside the brackets are performed and then only the real component of the division is considered. The same Taylor series expansion approach is used to analyze the discriminator function. The only difference is that all four error terms are included, (46).

$$R_{target} = \frac{L+1}{L-1} Re \left\{ \frac{V_c^* - V_b^*}{V_c^* + V_b^*} \right\} \frac{\delta R}{2} + R_{rgb} + \frac{\delta R}{2}$$
(45)

$$\frac{A_{c}[V_{c} + \delta_{x,c} + j\delta_{y,c}] - A_{b}[V_{b} + \delta_{x,b} + j\delta_{y,b}]}{A_{c}[V_{c} + \delta_{x,c} + j\delta_{y,c}] + A_{b}[V_{b} + \delta_{x,b} + j\delta_{y,b}]} = f(\delta_{x,c}, \delta_{y,c}, \delta_{x,b}, \delta_{y,b})$$
(46)

Equation (47) shows the first three terms of the Taylor series expansion. The partial with respect to $\delta_{y,c}$ results in an imaginary number. Since only the real component is considered, the contribution of $\delta_{y,c}$ to the series can be ignored. Likewise, the partial with respect to $\delta_{y,b}$ can also be ignored. With the terms contributed by $\delta_{y,b}$ and $\delta_{y,c}$ gone, the first order Taylor Series approximation of the second discriminator is identical to that of the first discriminator.

$$f(\delta_{x,c}, \delta_{y,c}, \delta_{x,b}, \delta_{y,b}) = \frac{A_c V_c - A_b V_b}{A_c V_c + A_b V_b} + \delta_{x,c} \frac{2A_c A_b V_b}{(A_c V_c + A_b V_b)^2} -\delta_{y,c} \frac{j2A_c A_b V_b}{(A_c V_c + A_b V_b)^2} + \dots$$
(47)

The variance of the range errors for the second discriminator can therefore be expressed with the same equation derived for the first discriminator. Theoretically, the discriminators should have the same performance.

C. Third Discriminator

Equation (48) shows the third discriminator function. It takes the absolute values of V_b^* and V_c^* and then performs the math operations of the discriminator. The absolute value operation does not lend itself to mathematical manipulation in this case. The third discriminators performance is evaluated empirically by simulation.

$$R_{target} = \frac{L+1}{L-1} \frac{|V_c^*| - |V_b^*|}{|V_c^*| + |V_b^*|} \frac{\delta R}{2} + R_{rgb} + \frac{\delta R}{2}$$
(48)

V. SIMULATION RESULTS AND VALIDATION

In order to both validate the analytical derivations of the first and second discriminators and study the absolute value discriminator, a simulation of the radar system was performed in MATLAB. Nine equally spaced points where chosen between R_{rgc} and R_{rgb} as shown in Figure 4. At each discrete point, the variance of the range error was computed by Monte Carlo simulation. Values of σ_{target} were first calculated for different signal to noise ratios. Then, a least-squares curve fit of the data was performed. The data was fitted to the function in equation (49). The value of k which gave the least-mean-squared error between the curve fit and the simulation results was determined.

$$\sigma_{target} = k \frac{\delta R}{\sqrt{SNR}} \tag{49}$$



Fig. 4. Discrete Positions Within Split-Gate Region

Figure 5 shows the results of the simulation for the first discriminator for the discrete position labeled number five

$$\sigma_{target}^{2} = \left(\frac{L+1}{L-1}\right)^{2} \left[\frac{1}{T} \frac{A_{c}^{2} A_{b}^{2} R_{b}^{2}}{(A_{c} R_{c} + A_{b} R_{b})^{4}} + \frac{1}{T} \frac{A_{c}^{2} A_{b}^{2} R_{c}^{2}}{(A_{c} R_{c} + R_{b} R_{b})^{4}} - \frac{R(\tau_{c} - \tau_{b})}{T} \frac{2A_{c}^{2} A_{b}^{2} R_{c} R_{b}}{(A_{c} R_{c} + A_{b} R_{b})^{4}}\right] \frac{(\delta R)^{2}}{2SNR}$$
(43)

$$\sigma_{target} = k \frac{\delta R}{\sqrt{SNR}} \tag{44}$$
where

$$k = \sqrt{\frac{A_c^2 A_b^2}{2T (A_c R_c + R_b R_b)^4} \left(\frac{L+1}{L-1}\right)^2 [R_b^2 + R_c^2 - 2R(\tau_c - \tau_b) R_b R_c]}$$

in figure 4 and no channel mismatch. The value of σ_{target} determined from the Monte Carlo simulation matches closely with the value derived earlier. The curve determined from the least-squares fit is also shown and accurately models the shape of the data. Figure 6 shows the results of the same simulation for the discrete position at the edge of the split-gate region, labeled number nine in figure 4. Again, the simulation results and analytical values match closely.



Fig. 5. Simulation results at the middle of the split-gate region with no channel mismatch for the first discriminator.



Fig. 6. Simulation results at the edge of the split-gate region for the first discriminator with no channel mismatch.

Figure 7 shows the predicted value of k and values determined from the least-squares fit of two different Monte Carlo simulations. The difference in the simulations is the initial seed in MATLAB's random number generator. The predicted and empirical values of k slightly diverge on the left side in the top graph. The opposite behavior is seen in the bottom graph. This behavior is attributed to the fact that the noise MATLAB generates is not truly random but based on the CPU state. Figure 8 shows how the value of k varies when the channels in the receiver are not match. For figure 8, $A_b = .8$ and $A_c = 1$.



Fig. 7. Predicted and Monte Carlo Values of k for the first discriminator with no channel mismatch.

Figure 9 shows the results of the Monte Carlo simulation for the second discriminator (45). The second discriminator produces the same values of σ_{target} as the first discriminator, as predicted.

Figure 10 shows the results of the Monte Carlo simulation for the third discriminator (48) with no channel mismatch. Figure 11 shows its performance with $A_b = .8$ and $A_c = 1$. Its performance is equivalent to the performance of the first two discriminators.

VI. CONCLUSION

In this paper the authors present three new contributions. First, we show that the variance of the range errors for a phase coded CW radar is a function of position in the splitgate region. Second, analytical equations are derived that



Fig. 8. Predicted and Monte Carlo values of k for the first discriminator with channel mismatch, $A_b = .8$ and $A_c = 1$.



Fig. 9. Predicted and Monte Carlo values of k for the second discriminator with no channel mismatch.



Fig. 10. Monte Carlo values of k for the third discriminator with the predicted values of k for the first discriminator. There is no channel mismatch.



Fig. 11. Monte Carlo values of k for the third discriminator with the predicted values of k for the first discriminator. The channel mismatch is .2, $A_b = .8$ and $A_c = 1$.

accurately relate the variance of the range errors to SNR and channel mismatch. The third contribution is a comparison of three different discriminator functions. All three discriminators are shown to produce similar results. These contributions are significant because they allow the range errors encountered in split-gate tracking to be better modeled. Correct modeling of the range errors is important because it accelerates the convergence of the track filter.

Future work in the area would include comparing the results in the paper to actual data. By using real data, the results shown in this paper could be validated. Using real data would allow the performance of the track filter to be investigated. How fast the track filter converges when using our results can be compared to how fast it converges when it assumes the variance of the range error is constant through the split-gate region.

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